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**GREEK MATHEMATICS IN
PLATO'S *PARMENIDES***

Today...

- ◎ STEM sciences and the Humanities are taught separately.
 - Each discipline has grown so large that scholars, professors and students specialize in their studies
 - We focus on one kind of problem and become a specialist
 - We become isolated in our disciplines

Ancient Greece (5th Century BCE to 4th Century BCE)

- ◎ Inquiries into nature and number were combined with philosophical arguments, literature and poetry.
 - Pythagorean cosmogony and religious beliefs were combined with mathematical discoveries of number
 - Parmenides wrote his poem about what we can know based on the structure of reality
 - Zeno defended Parmenides by paradoxes using mathematical concepts of the finite and the infinite

The Bigger Picture ...

- ◎ Plato's *Parmenides* is written at a juncture.
 - Pythagoreans, “all things are numbers”/pluralist pre-Socratics (5th Century BCE)
 - Eleatic response to pluralistic metaphysics, “all that is, is one”.
 - Response from the pluralists, “if everything is one, then many absurdities follow”.
 - Zeno's defense of Parmenides
 - Post-Eleatic Pythagoreans/Platonic Metaphysics (4th Century BCE)
- Let's learn a little about Pythagoras

Proclus (5th Century, C.E.)

- ‘Keeping count’ started with the Phoenicians for bookkeeping grain stores.
- Geometry “land measure” began with the Egyptians to measure the land to levy taxes against it.

Ancient Greek Mathematical Concepts

⊙ even/odd

- *Artios*: that which can be divided into two equal parts
- *Perittos*: that which cannot be divided into two equal parts or that which differs from the even by a unit

⊙ *arithmos/monas*

- ἀριθμός : a limited multitude
- μονάς : “unit”, the least definite thing “of all possible partitions” (Klein 1967:42)

Szabó on Form or *Eidos*

- ⦿ The Greek word for ‘to define’ (*ὀρίζεσθαι*) ... means to *mark off*.
- ⦿ A definition was intended to mark off the Form or *Eidos* of an object from that which it *was not* and in this way secure the consistency of the Form in question.

Pythagoras of Samos (5th BCE)

- “Pythagoras himself seems to have been one of those rare figures in history who are at once great religious leaders and pre-eminent scientists.” (Raven, 1948: 175)

Pythagoras of Samos

- Believed in a fundamental dualism:

Limit	Unlimited
Unity (one)	Plurality (dyad)
Rest	Motion
Goodness	Evil

Which led to an interesting story about how the world was made.

Pythagoras of Samos

- ① “Cosmogony [for Pythagoras] consists in the progressive inhalation and limiting of the latter principle by the former. The outcome of the process is a plurality of sensible things which, being sums of spatially extended units kept apart by the void, are equal to numbers.” (Raven, 176)

Pre-Eleatic Pythagoreans

- ⦿ Like all thinkers, they were interested in what makes up the world and how the world is caused.
- ⦿ They were the first to claim that the structure of reality consists of ἀριθμοί, “numbers”
- ⦿ Their cosmology is a combination of matter and number

Pythagorean Cosmogony

- ⦿ “...this principle of Unity or Limit was conceived as having started the whole Pythagorean cosmogony by ... injecting ‘the first unit with magnitude’ like a seed into the womb of the Unlimited; and that first unit, which began forthwith to breathe in and limit the Unlimited, proceeded to generate, by the successive introduction of intervals of the Unlimited into its own nature, first the line, then the plane and finally the solid.” (Raven 1948:115)

Pre-Eleatic Pythagorean Cosmology

Point

Line

Plane

Solid

Early Pythagoreans

- ⦿ Applied ἀριθμός “a limited multitude” to magnitudes (lines, planes and solids) in addition to units.
- ⦿ The problem of incommensurability did not arise with the early Pythagoreans. (Knorr 1975)
- ⦿ (my conjecture) They held that square numbers and square plane numbers shared the same properties.
- ⦿ Foundational problem: the relation between the one and the many / limit and unlimited

Parmenides objected to this pluralistic view of reality



Parmenides from Elea

- Born 515 BCE (roughly)
- From Elea, now Ascea in Southern Italy
- Objected to a pluralistic ontology
- Argued that what was 'real' was unchanging

Examples of his thought...

Parmenides, b7

...hold your thought back from this route of inquiry
and do not let habit, rich in experience, compel
you along this route to direct an aimless eye
and an echoing ear and tongue
but judge by reasoning (*logos*) the much
contested examination spoken by me.

Tr. Richard McKirahan

Parmenides, b8

Just one story of a route
is still left: that it is.

On this there are signs
very many, that what –is
is ungenerated and imperishable,
a whole of a single kind,
unshaken, and complete...

Tr. Richard McKirahan

Parmenides, b8

...Therefore it has been named all things
that mortals, persuaded that they are true,
have posited
both to come to be and to perish, to be and not,
and to change place and alter bright color.

But since the limit is ultimate, it [namely, what-is] is complete
From all directions like the bulk of a ball well-rounded from all
sides
Equally matched in every way from the middle; for it is right
for it to be not in any way greater or lesser than in another. ...

Tr. Richard McKirahan

Parmenides, b8

From what-is-not

I will allow you neither to say nor to think:

For it is not to be said or thought that it is not.

Tr. Richard McKirahan

Zeno's work is a defense of Parmenides's monism

Plato's *Parmenides*

- Tells us that Zeno's work was created to defend his teacher, Parmenides, against the objections from the pluralists.

Árpád Szabó on Zeno

According to Simplicius, Zeno was engaged in contrasting one *hypothesis* with another:

- ⊙ ἡ ὑπόθεσις ἡ λέγουσα πολλά ἔστιν
‘the *hypothesis* which states that what exists is *many*’ with
- ⊙ ἡ τοῦ ἓν εἶναι ‘the *hypothesis* ‘which states that what exists is *one*.’

Zeno's paradoxes...

- ⦿ Argued that motion, space and time, if they consist of pluralities, led to contradictions.
- ⦿ For example...

Zeno's paradoxes...

1. If things are many, then they are finite in number, neither more nor less than they are.
2. If things are many, they must be infinite in number, for there are always things between that which exists.
3. If a thing exists, it either has magnitude or it does not.
4. If it has no magnitude, then it cannot be made larger or smaller, nor can it make something larger or smaller. Therefore, if it has no magnitude, it doesn't exist.

Zeno's paradoxes...

5. If something has magnitude, then its parts must have size and bulk, and these in turn, have a distance from one another. And each part of a part must have a size and distance from each other, and so on.
6. No part, however small, can be the ultimate part, nor will any part lack parts of its own.
7. Therefore, if things are many, they must be both large and small. So small as to have no size, so large as to be infinite.

'reductio ad absurdum' is dialectical reasoning
This format is also known as Indirect proof

Mathematics and Dialectic

- ⦿ Dialectic came before mathematics (Szabó)
- ⦿ Aristotle claims that Zeno invented the method.
- ⦿ Dialectic is a *debate*.
 - αἴτημα | *aitēma* (a ‘request’ or ‘demand’) synonyms:
 - ὑπόθεσις | *hypothesis*
 - ὑποκείμενον | *hypokeimenon*

But it is a debate about definitions

Mathematics and Dialectic

- ◎ Szabó (1978, p. 269):

A joint investigation could not be based on an assumption or *hypothesis* unless *both* participants agreed to it. Hence one of them had to *ask* for the agreement of the other. An agreed definition could be called *homologēma* or *hypothesis*.

Ex: *Meno* 86e3; *Theaetetus*; *Parmenides*

Mathematics and Dialectic

⦿ *Mathē/mathēmata*

- Μάθημα “learning matter”
- Μάθησις “study, discipline”

⦿ Μαθηματικά “mathematical objects”

Plato is writing at the time where questions about reality – what there is, what is *real* – and what can we know about these objects intersected with mathematical inquiry.

Luc Brisson

An interpretation of Plato's *Parmenides* must address these three questions:

1. What weight should be lent to the staging around Parmenides and Zeno?
1. How should the critique of Forms, in Part I be interpreted? [T1, T2]
1. How do the two parts of the dialogue relate to each other? [T1-T2: T3]

Plato's *Parmenides*, Part II

- ⦿ Deductions are a dialectical exercise
 - Starting with contrary hypotheses of The One
- ⦿ The One is the subject of every deduction
- ⦿ The One is a Form, but does not stand for every Form
- ⦿ The 'exercise' helps determine what can be said of the One and its instantiations

Plato's response to Parmenides is in an indirect proof

- If we posit a Form for every object, sensible or abstract, we arrive at contradictions or infinite regresses.
- If we don't posit any Forms, we can't have knowledge of the world

Plato's response to Parmenides is in an indirect proof

- ⦿ There must be at least one Form that exists.

Plato's *Parmenides*, Part II

- ⦿ Since the One *is*
- ⦿ And everything participates in the One
- ⦿ It follows that
- ⦿ If an object participates in the One and another Form, then the object is an *arithmos*.

So what does this tell us?

Plato's *Parmenides*, Part II

- ⦿ The deductions tell us how the world is affected by a single form, the One.
- ⦿ We learn a general account of the Form-particular relation.
- ⦿ This account is not different from the account we first received from Plato in the *Phaedo*. [T1 : T4]

Answer to the three questions

- Zeno and Parmenides frame the solution to understanding Plato's metaphysics via the dialectic (i.e., mathematical reasoning)
- Criticism by Parmenides, that Forms lead to an infinite regress on a particular reading, should be rejected
- Socrates shows us how by using the dialectic in Part II of the *Parmenides*

Conclusion

- The theory of forms is a precursor to ancient Greek mathematics, founded upon an Eleatic account of Being and Post-Eleatic Pythagorean dualism.
- To understand the Form-particular relation, one must understand the Greek concept of *arithmos* and its properties.

Conclusion

- ⦿ A Form defines the characteristic of objects by providing a limit/boundary to the objects.
 - To understand a Form is to understand it as a unity. (Parmenides B8; Plato's *Phaedo* 78d-e, Symposium 211a-d)
- ⦿ Objects are given their characteristics by their participating in Forms.
 - To understand a plurality is to understand it as an *arithmos*, 'number'

Thank you!

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